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ABSTRACT

The primary objective of this research was to examine the relationship between student achievement in mathematics and pedagogical approach used by middle school mathematics teachers in the United States who participated in the Third International Mathematics and Science Study. In this research, student achievement was explored at the item, rather than test, level with the thought that differences might be found only at this micro level. It was hypothesized that middle school mathematics students whose teachers utilized a more student-centered, or constructivist, pedagogical approach would have a higher probability of obtaining the correct answer to mathematics items that measured conceptual, rather than procedural, understanding. This hypothesis was explicitly tested using differential item functioning analyses. Results support the hypothesis, although not as strongly as had been expected. An appendix contains the teacher survey. (Contains 3 tables, 1 figure, and 10 references.) (Author/SLD)

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Running head: EFFECT OF DIFFERENT PEDAGOGICAL APPROACHES

The Effect of Different Pedagogical Approaches on Mathematics Students' Achievement

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Abstract

The primary objective of this research was to examine the relationship between student achievement in mathematics and pedagogical approach used by middle school mathematics teachers, in the United States, who participated in the Third International Mathematics and Science Study. In this research, student achievement was explored at the item, rather than test, level with the thought that differences might be found only at this micro level. It was hypothesized that middle school mathematics students whose teachers utilized a more *student-centered*, or constructivist, pedagogical approach would have a higher probability of obtaining the correct answer to mathematics items that measured conceptual, rather than procedural, understanding. This hypothesis was explicitly tested using differential item functioning analyses. Results supported the hypothesis, although not as strongly as had been expected.

Introduction

For decades the educational community has debated the strengths and weaknesses of a *teacher-directed* approach to instruction as opposed to a *student-centered* approach to instruction. As early as 1900, Dewey advocated the need for schools to consider the experiences students brought with them to the classroom and be "active centers of scientific insight into natural materials and processes" (Dewey, 1900, p. 19). He criticized traditional education for emphasizing the rote memorization of rules and symbols while treating students as passive absorbers of information, rather than active vital beings whose knowledge is constantly being constructed and transformed. Soon after, Thorndike (1906) equated teaching with the ability to enable students to recall facts and information from memory and believed this was best accomplished by repetitive practice. He stated, "the active recall of a fact from within is, as a rule, better than its impression from without"(Thorndike, 1906, p.123). While admitting that the teacher directed learning was being criticized for not promoting student understanding and not teaching students how to think, he argued that in some cases the best way to ensure a student was thinking was to give the student facts to think about.

More recently, with the publication of the *Curriculum and Evaluation Standards* (NCTM, 1989) and the *Professional Standards for Teaching Mathematics* (NCTM, 1991) [Standards], there is an implicit belief among most educators that a *student-centered* approach to teaching mathematics is superior to the more traditional "talk and chalk" or *teacher-directed* approach to teaching. These documents reflect what most mathematics educators feel empirical research supports in terms of what it means to know and understand mathematics, as well as how this knowledge of mathematics can best be acquired within a learning environment. According to the Standards "*what* a student learns depends a great deal on *how* he or she has learned it" (NCTM, 1989, p .5). If this is true, then students coming from different learning environments should theoretically possess different types of knowledge and understanding. For example, a student who learns mathematics in a *teacher-directed* classroom, from a teacher who believes that teachers must direct students' thinking should possess a different type of knowledge and understanding than a student who learns mathematics in a *student-centered* classroom, from a

teacher who believes students should construct their own knowledge base by exploring, conjecturing, and reasoning.

In the case of *teacher-directed* learning, students are thought of as a tabula rasa where learning is characterized as passive and receptive (Dengate & Lerman, 1995). When students are taught in this fashion, it is logical to assume that their knowledge is somewhat limited to what was shown to them and what they remembered. These students should have acquired only a procedural understanding of the mathematics being taught, knowing *what* to do to solve particular mathematics problems but not *why* they were doing it. Although some students taught in this manner would probably be capable of generalizing what they learn, enabling them to reason mathematically and creatively problem solve, the majority of students probably would not.

In the case of *student-centered* learning, students' knowledge would logically seem less limited, for even if a student did not recall how to solve a particular type of problem, these students would have learned they are capable of figuring it out on their own. These students should have acquired a conceptual understanding of the mathematics being taught, knowing not only *what* to do but *why* they were doing it. The conceptual understanding acquired by these students should enable them to apply their knowledge in new mathematical situations. In other words, the underlying distribution of conceptual understanding, including mathematical reasoning and problem solving abilities, for these students should be higher than the underlying distribution of students who are taught in a more teacher directed manner.

One of the mainstream explanations for the occurrence of differential item functioning is that items are actually measuring more than one ability while the groups considered differ in their underlying distributions on these abilities, yet only one ability is reported (Ackerman, 1992; Ackerman & Evans, 1994). If some of the mathematics problems considered actually measure conceptual understanding, in addition to procedural understanding, then given the theoretical considerations, differential item functioning should occur for students who are grouped based on the pedagogical approach used by their teacher. Specifically, those students taught from a more *student-centered* environment should have a higher probability of obtaining the correct answer to items that are measuring conceptual understanding, in addition to procedural understanding.

Much time, energy, and money is invested in pre-service and in-service teacher training, as well as in curriculum changes, which promotes the belief that a *student-centered* learning

environment is superior to a *teacher-directed* learning environment. However, presently and historically little empirical evidence can be found which supports this belief. This intent of this research is to explore the relationship between student achievement in mathematics and pedagogical approach used by middle school mathematics teachers. This will help to provide some much needed empirical evidence as to what learning environment is most conducive to student learning and understanding in mathematics

Methodology

Participants

The students in this study consist of Population 2 students, from the USA, who participated in the Third International Mathematics and Science Study [TIMMS]. "Population 2", a term used by those who developed TIMMS refers to 13 year old students. These students were in the 7th or 8th grade at the time of testing. More specifically, Population 2 is defined as the two adjacent grades that will maximize coverage of 13 year old students (Martin & Kelly, 1996).

Instrumentation

Measure of Mathematical Ability

All of the multiple choice mathematics items from the TIMMS test administered to Population 2 students were used as the measure of mathematics ability. This test consists of eight different booklets, each consisting of 40 items pertaining to either mathematics or science. Only booklets one through seven were used. All mathematics items were classified in terms of content, performance expectations, and context. The content areas covered by items in the test include fractions and number sense, algebra, data representation, analysis and probability, measurement, and proportionality. Performance expectations include knowing, using routine procedures, problem solving, mathematical reasoning, proportionality, and communication (Robitaille et. al., 1993). Table 1 displays the number of items in each booklet which were classified into each content area.

Table 1

Number of Mathematics Items in Each Booklet by Content Area

	Booklet							
Content area	1	2	3	4	5	6	7	8
Fractions and number sense	11	10	11	10	10	11	11	14
Geometry	5	6	6	3	6	4	5	6
Algebra	8	5	6	8	4	6	6	9
Data representation, analysis and probability	5	4	4	6	7	6	7	5
Measurement	5	5	6	4	6	4	4	3
Proportionality	3	3	4	3	6	2	4	4

Test Design

A variant of matrix sampling was used in the TIMMS test design. In other words, not all items were administered to all examinees. Specifically, a subset of items was assigned to individual students so as to produce reliable estimates of the populations' performance on all items, even thought no student has responded to all of the items (Martin & Kelley, 1996). This approach allows students to respond to a much smaller number of items than the total number of items in the pool, which, including both mathematics and science items, is equivalent to 530 items. This approach eliminates the negative impacts associated with requiring students to spend the number of hours necessary to complete all the items, such as fatigue, decreased motivation, and reduced participation rates (Johnson, 1992). It has been estimated that it would take almost seven hours of testing for one student to complete all the items (Martin & Kelley, 1996). Moreover, the approach used also results in more efficient estimations of performance in subpopulations, in terms of errors of the estimates (Johnson, 1992).

The TIMMS item pool was divided into 26 mutually exclusive clusters, each labeled by a different letter from the alphabet. Items in clusters A through H consist of mathematics and science multiple-choice type items. Items in clusters I through R consist of mathematics and science multiple-choice type items, as well as short-answer items. Items in clusters S through U consist of only mathematics extended-response type items. Items in cluster V consist of both mathematics and science multiple-choice type items, as well as short- answer and extended-response items. Clusters W through Z consist of only science extended-response type items (Marin & Kelley, 1996). These clusters were then arranged into eight booklets, each of which contains up to seven item clusters. Cluster A, the core cluster, appears in the second position in

every booklet, whereas the remaining clusters each appear in as few as one booklet, or as many as three. Booklet 8 consists only of items not present in other booklets and therefore was not analyzed in this study.

The international median KR-20 coefficients across the eight test booklets for seventh grade students ranged from 0.91 in Hong Kong to 0.75 in Iran. The international median KR-20 coefficients across the eight test booklets for eighth grade students ranged from 0.91 in Bulgaria to 0.73 in Kuwait. The median KR-20 for the United States was 0.89. The international median, which is the median of reliability coefficients for all countries, is 0.86. Table 2 provides the internal reliability for the subset of participating examinees used in this study for each of the seven booklets used.

Table 2

KR-20 Coefficients for Booklets 1 Through 7

Booklet	Sample size (n)	KR-20
1	984	0.86
2	986	0.86
3	990	0.85
4	987	0.87
5	993	0.87
6	960	0.86
7	980	0.88

Teacher Questionnaire

A survey, consisting of four sections, was administered to all participating teachers. The first section gathered demographic information of teachers, including the level of mathematics they teach, how often they teach, how much time they spend on activities related to teaching, beliefs about mathematics, familiarity with Standards documents, and how much time they are able to interact with other teachers. The second section consists of questions specific to the class being tested including, textbook used, limitations to teaching, calculator use, planning, as well as classroom activities. The third section consists of a set of exercises pertaining to various mathematical topics and questions teachers whether or not they have taught or will teach the content area covered by the given exercises to their class during the school year. The fourth section is designed to measure teachers' beliefs about how mathematics is best taught.

Sixteen questions from the teacher survey were used to create a measure designed to differentiate teachers with opposing pedagogical beliefs. Appendix A contains the measure in its entirety. The items used were scored such that a low number represented a teacher directed pedagogical belief and a high number represented a student directed pedagogical belief. The lowest possible score a teacher could obtain was 16, while the highest possible score a teacher could obtain was 60. Actual scores ranged from 22 to 53. The reliability of this measure was 0.80. Figure 1 illustrates the distribution of scores on this measure. Fifteen of the questions used were Likert-type survey questions while one of the questions used was a ranking task. Of the 527 teachers administered the survey, 161 were eliminated because they either did not complete the survey at all, or only completed part of the survey. Many of these teachers may not have completed the items used because they taught science, rather than mathematics because only 374 mathematics classes were tested. This would imply that only 8 mathematics teachers were eliminated from the study.

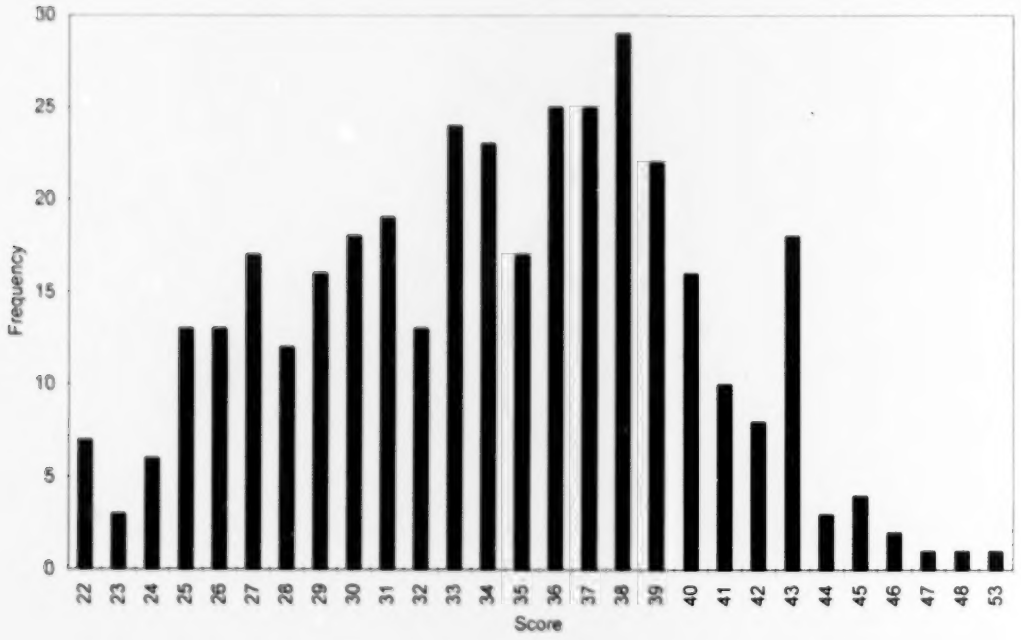

Figure 1. Distribution of scores on the teacher survey.

Questions 1 through 15 used were Likert-type survey questions with either three or four response options. Questions 1 through 4 pertain to how often a teacher asks students to do some particular task. A response or 'never or almost never' was scored as 1, 'some lessons' was scored as 2, 'most lessons' was scored as 3, and 'every lesson was scored as 4. Questions 5 through 7 pertain to how important a teacher believes it is for students to possess certain mathematical skills in order for students to succeed in school mathematics. A response of 'not important was scored as 1, 'somewhat important' was scored as 2, and 'very important' was scored as 3. Questions 8 through 15 pertain to the type of homework teachers assign. Those teachers who stated they did not assign homework were eliminated from the sample. A response of 'never' was scored as 1, 'rarely' was scored as 2, 'sometimes' was scored as 3, and 'always' was scored as 4. It was not necessary to reverse-score any of the Likert-type items used. Missing responses for teachers who completed the entire survey were given the modal response for these Likert-type items.

Question 16 was a ranking task. This question presented teachers with a classroom situation and asked teachers to rank order them, placing a '1' in the box next to the approach they believed was best. This task was scored 1 if a teacher chose approach 'a', 'b', or 'c', as the best, approach, 2 if a teacher chose approach 'd' or 'e' as the best approach, and 3 if a teacher chose approach 'f' as the best approach. Teachers who ranked 'a', 'b', or 'c', as the best way to approach the topic can be considered to use more of a traditional pedagogical approach, because all of these approaches have the teacher telling or showing the students something. Although teachers who ranked 'd' or 'e' as the best way to approach the topic have the students working with the teachers, it is unclear how they will be working together, in terms of if the class is more student centered or teacher centered. However, it would seem that these approaches are more teacher-directed because the teacher is imposing their solution strategies onto the students. Those teachers who chose approach 'f' as the most ideal approach can be considered to be utilizing a more constructivist approach as they are allowing the students to try and solve the problem on their own.

The 16-item chosen from the TIMMS teacher questionnaire to measure teachers' pedagogical beliefs was used to group students. Student test data was linked to teacher questionnaire data using classroom identification numbers. Therefore, students whose teachers were eliminated from the sample were also eliminated from the sample. A stringent criterion

was set to label teachers as "constructivist". Only teachers who scored above the 90th percentile were classified as "constructivist" teachers. For all booklets, approximately 89% of teachers obtained a score below 41 on the teacher measure. Therefore students whose teacher obtained a score above 41 on the teacher measure were considered as being from the "constructivist" or minority group, and students whose teacher obtained a score about 41 were considered the "teacher-directed" or majority group.

Analyses

Due to the complexity of the test design, analyses were conducted on each booklet separately. This strategy permits each item used in the mathematics achievement test of TIMMS to be used. It also allows cross-validation of results across different groups of examinees for items that appear in more than one booklet. These analyses were conducted using SIBTEST (Shealy & Stout, 1993) in an exploratory fashion to detect items which students from a *student-centered* classroom have a higher probability of obtaining a correct answer. Due to the design of the TIMMS study, it was possible to achieve conflicting results in the DIF analyses, such that an item that exhibited DIF in one booklet did not demonstrate DIF from another booklet. However, since the statistic provided by SIBTEST comes from a normal distribution with known variance, it was possible to average the test statistics provided by each analysis over the same items. Specifically, since the statistic calculated by SIBTEST is normally distributed, the mean of these statistics is also normally distributed. Therefore, the standard deviation of the mean of the three test statistics, obtained from testing the same item from three different booklets, can be obtained using normal distribution theory. This in turn allows one to calculate a test statistic for the mean of these test statistics, as the following proof demonstrates.

Let β come from a normal distribution with variance σ^2, then

$$\mathrm{Var}(\bar{\beta}) = \mathrm{Var}\,\frac{(\beta_1 + \beta_2 + \ldots + \beta_n)}{n}$$

$$= \frac{1}{n^2}(\mathrm{Var}(\beta_1) + \mathrm{Var}(\beta_2) + \ldots + \mathrm{Var}(\beta_n))$$

The variance of β_i is not directly provided by SIBTEST; however, since the β statisticand the associated z-score are given the standard deviation is easily obtained by dividing the β statistic by the z-score associated with it. Squaring the standard deviation provides the variance of each

test statistic. In all cases when an item appeared in more than one booklet this approach was used.

Results

Table 3 illustrates the results of the differential item functioning analyses for the focus items that appeared in more than one booklet. None of the other items that appeared in only one booklet showed a statistically significant amount of DIF. Items labeled A1 through A6, appeared in all seven booklets (n = 7), while all other items appeared in only three booklets (n = 3). Unfortunately, the items that appear in more than one booklet are restricted making it impossible to include them in this paper.

Table 3

Average Test Statistics for Items Appearing in More than One Booklet

Item	$\bar{\beta}$	σ^2	z-score
A1	-0.057	0.022	-2.577*
A2	0.026	0.022	1.156
A3	-0.013	0.026	-0.490
A4	-0.018	0.021	-0.843
A5	-0.010	0.025	-0.383
A6	0.032	0.022	1.463
B1	0.025	0.026	0.939
B2	0.014	0.032	0.427
B3	-0.011	0.029	-0.365
B4	-0.009	0.025	-0.362
B5	0.004	0.027	0.158
B6	-0.024	0.024	-0.992
C1	0.046	0.029	1.592
C2	0.030	0.026	1.131
C3	-0.094	0.028	-3.407*
C4	-0.003	0.033	-0.092
C5	-0.010	0.030	-0.341
C6	-0.021	0.027	-0.777

* $p < 0.05$

(table continues)

Table 3 (continued)

Average Test Statistics for Items Appearing in More than One Booklet

Item	$\bar{\beta}$	σ^2	z-score
D1	0.014	0.041	0.331
D2	0.029	0.025	1.157
D3	0.016	0.025	0.621
D4	0.000	0.029	0.000
D5	-0.013	0.025	-0.519
D6	0.005	0.030	0.180
E1	0.002	0.031	0.054
E2	0.007	0.030	0.243
E3	0.072	0.033	2.207*
E4	0.003	0.033	0.080
E5	-0.007	0.027	-0.269
E5	0.028	0.025	1.116
F1	-0.018	0.028	-0.650
F2	-0.021	0.033	-0.652
F3	0.043	0.027	1.592
F4	0.009	0.025	0.353
F5	-0.008	0.029	-0.277
F6	-0.043	0.025	-1.742*
G1	-0.006	0.030	-0.189
G2	0.007	0.026	0.286
G3	-0.041	0.029	-1.405
G4	0.009	0.026	0.352
G5	0.042	0.028	1.523
G6	-0.001	0.027	-0.024
H1	-0.018	0.027	-0.655
H2	0.029	0.023	1.250
H3	0.022	0.021	1.040
H4	0.005	0.030	0.168
H5	-0.033	0.029	-1.117
H6	-0.033	0.025	1.293

* $p < 0.05$

Discussion

Several items did show a statistically significant amount of DIF in favor of the focal group and these items were considered to be measuring more of a conceptual understanding of mathematics. For example, one of the items that showed a statistically significant amount of DIF

asked examinees to determine how many more pieces needed to be shaded in a given figure so that a certain fraction of the figure was shaded. However different denominators were used in the symbolic and pictorial representations of the fraction, implicitly requiring examinees to find a common denominator for the two different representations in order to successfully complete the problem. This would imply that examinees had to know not only *how* to convert fractions so that they had the same denominator, but *why* one would want to do so. Furthermore, the reason why one needed to find a common denominator in this situation was not the standard or typical reason why, because no computation was needed to complete the problem.

A similar argument could be made for all of the other items that showed a statistically significant amount of DIF in favor of students taught in a more *student-centered* environment. However, this same argument could probably hold for some of the other items that did not exhibit a statistically significant amount of DIF but were thought to be measuring more of a conceptual understanding of mathematics, including mathematical reasoning. Furthermore, one item was found to be functioning differentially in favor of students who were taught in a more *teacher-centered* environment. So what does all this mean? It would seem that different pedagogical approaches have some effect on the way students view items, but not as drastic as had been expected. There are several different ideas that come to mind as to why this might be the case.

First of all, one of the biggest limitations of this study is the method used to group examinees. Although the measure was fairly reliable, it relies on teachers' self report as to what they are doing in the classroom. This report may or may not be accurate. In a qualitative study conducted by TIMMS researchers, it was found that few, if any, mathematics teachers in the United States truly teach in a constructivist manner. It is possible that the grouping method utilized affected the results, although it was the best that could be done with the given data. Secondly, it is difficult to determine just how much of an effect one teacher has on their students. Ideally, it would be nice to think that one good teacher who teaches mathematics to encourage conceptual understanding can eradicate any ill effects of previous teachers who focused more on procedural understanding of mathematics. However this is probably affected by many other factors.

In conclusion, the main purpose of this research was to explore the relationship between pedagogical approach used by mathematics teachers and student achievement and to determine if items which were measuring more of a conceptual understanding of mathematics functioned differentially for examinees who were taught from contrasting pedagogical approaches. Theoretically it makes sense that this should be the case. However these results do not conclusively show this to be true or false. While several items did exhibit DIF in favor of students who were grouped into the *student-centered* group, many items did not. Furthermore, due to the grouping methodology used it is uncertain how contrasting the pedagogical approaches used by teachers in the two groups actually were. Future research is warranted to further explore these issues.

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APPENDIX A

TEACHER SURVEY

In your mathematics lessons, how often do you usually ask students to do the following?

1. explain the reasoning behind an idea	never or almost never	some lessons	most lessons	every lesson
2. represent and analyze relationships using tables, charts, or graphs	never or almost never	some lessons	most lessons	every lesson
3. work on problems for which there is no immediately obvious method or solution	never or almost never	some lessons	most lessons	every lesson
4. write equations to represent relationships	never or almost never	some lessons	most lessons	every lesson

If you assign mathematics homework, how often do you assign each of the following kinds of tasks?

5. reading in a textbook or supplementary material	never	rarely	sometimes	often
6. writing definitions or other short writing assignment	never	rarely	sometimes	often
7. small investigation(s) or gathering data	never	rarely	sometimes	often
8. working individually on long term projects or experiments	never	rarely	sometimes	often
9. working as a small group on long term projects or experiments	never	rarely	sometimes	often
10. finding one or more uses of the content covered	never	rarely	sometimes	often
11. preparing oral reports either individually or as a small group	never	rarely	sometimes	often
12. keeping a journal	never	rarely	sometimes	often

To be good at mathematics at school, how important do you think it is for students to...

13. be able to think creatively	not important	somewhat important	very important
14. understand how mathematics is used in the real world	not important	somewhat important	very important
15. be able to provide reasons to support their solutions	not important	somewhat important	very important

16.

Each year many teachers must help their students learn to solve problems such as "Juan was able to run 1.5 kilometers in 5 minutes. If he was able to keep up this average speed, how far would he run in 12.5 minutes?" If you needed to help your class solve such problems, what approach or sequence of approaches do you believe would best help students learn?

Place a '1' in the box in the right-hand margin next to the approach you believe to be the best. If you believe other approaches would also be acceptable, place a number in the box next to each one indicating the order in which you would consider using it. You need not choose more than one approach. Write zero in the box for any approach you do not consider acceptable.

	Teaching Approach
a	I would present a general graph such as this because an understanding of graphs with a constant ratio of change in x to change in y is one important mathematical tool for solving problems like this one.
b	I would present the method of using proportional equations to solve this problem, as in: $\frac{1.5}{5} = \frac{x}{12.5} \rightarrow 5x = (1.5)(12.5) \rightarrow x = 18.75/5 = 3.75$ km After presenting other examples of this problem, I would assign practice exercises to students.
c	I would use the method suggested by the textbook for dealing with problems of this type, carrying out the strategy suggested by the textbook.
d	I would work with students to develop a reasonable graph for this *specific* problem, such as the one to the right and then work with students on using the properties of graphs like this one to find a numerical solution to the problem.
e	I would have students use a calculator to find pairs of numbers that related to how long a person has run at a constant average speed to how far that person has traveled. I would then have the students use these pairs of numbers to study how to determine the distance a person running at constant average speed would travel in a given time.
f	I would divide the class into several groups and have the students in each group work together on the problem until each group found a method for solving the given problem and then found a method that would work for similar problems.